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Augmented reality and Mathematics Education: Application of augmented reality on geometric shapes

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ABSTRACT

The application of augmented reality on geometric shapes revolutionizes the learning of mathematics by offering an interactive and immersive approach. Using this technology, students can visualize and manipulate geometric shapes in their real-world environment, making the abstract concepts of geometry more concrete and accessible. Augmented reality also allows for personalization of learning by adapting activities to the individual needs of students. By integrating augmented reality into the teaching of geometric shapes, educators can stimulate students' interest in mathematics and promote the development of essential skills such as problem solving and spatial thinking.

Keywords: augmented reality, learning of mathematics, interactive, immersive, geometric shapes.

1. Introduction

Augmented reality learning represents a significant evolution in contemporary educational methods, providing learners with an immersive and interactive experience that transcends the limits of traditional learning environments. This innovative technology allows virtual elements to be superimposed on the real world, creating unique opportunities to explore and understand abstract concepts in a tangible and engaging way. In this context, the application of augmented reality on geometric shapes emerges as a remarkable example of the integration of this technology in the teaching of mathematics. By allowing students to visualize and manipulate geometric shapes in their physical environment, augmented reality transforms complex mathematical concepts into concrete experiences, making it easier to understand and assimilate the fundamental principles of geometry. This introduction explores the impact of the application of augmented reality on learning geometric shapes, highlighting the benefits and opportunities it offers to enrich students' learning experience [1].

To evaluate the effectiveness of this technology, an experimental part of this study implemented a method to evaluate the effectiveness of the application of augmented reality on geometric shapes among students. The analysis shows that contextualizing information in augmented reality helps students understand geometric surface calculations faster than paper guides [2].

Furthermore, we offer recommendations to improve the relevance of the information provided by augmented reality and identify the types of operations that can greatly benefit from augmented reality.

Finally, based on the AR data model, we propose an ontology to integrate augmented reality content creation to train and guide students to fully understand the surface and area of geometric shapes.

Hypothetically increased reality could be a geometry it is for this reason after an presentation to epipolar geometry and the explanation of the strategies utilized, we are going follow the steps that lead to the improvement of a 3D show and after that move on to viable thinks about for the realization of our application. We are talking around epipolar geometry between photographs taken with a video framework one (camera) from a diverse point of view. So the plane going through a point that has a place to the environment and the image points is called epipolar. By following all these planes, we note that they meet at one point within the pictures, the epipoles (e1 and e2).

Computer vision accounts for the connections between scene and photo through the utilize of sensors and scientific models. This movement centers on picture handling, visual information translation and utilize. The geometric visual information will permit creating a interface with the visual controls, by the thought of [3]:

- Projection in a photo of a point of the scene,
- Connections between the diverse projections of the scene,
- Geometric recreation of the scene from projections in different photographs.

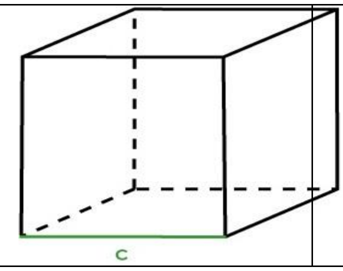
The applications are differing. Their divisions are basically [4]:

- Acknowledgment of shapes,
- modeling of situations,
- Dimensional control,
- expanded reality and help in picture blend or representation,

2. Problematic

Application of augmented reality on geometric shapes (Figure.01, 02, 03):

Cube



The area of the cube The volume of the cube

AB= c2

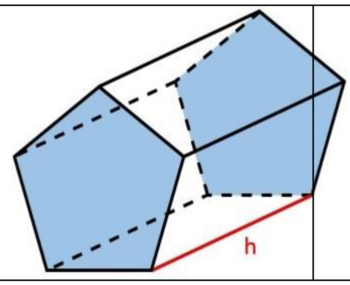
AL=4c2

V=c3

AT=6c2

Fig.01: theoretical study on paper, classic method

Prism



The area of the prisms The volume of the prisms

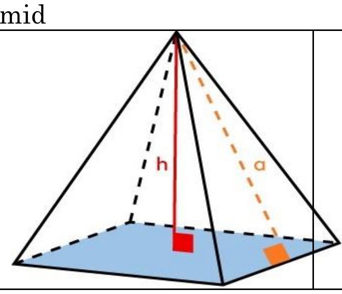
Ab= formula associated with the figure

AL=Pb×h V=Ab×h

AT=AL+2Ab

Fig.02: theoretical study on paper, classic method

Pyramid



The area of the pyramids The volume of the pyramids

Ab= formula associated with the figure

$AL = \frac{Pb \times a}{2}$ $V = \frac{A_b}{3} \times h$

AT=AL+Ab

Fig.03: theoretical study on paper, classic method

3. Methodology

3.1. Epipolar geometry

The epipolar geometry permits to create a geometric interface between 2 pictures of the same scene, it inspires the commitments connecting the perceptions of the same scene watched by one or two cameras, famous C1 and C2 (figure.04). These commitments are live related with the relative situating between the two cameras but are totally free of the structure of the scene. It is vital to keep in mind that when moving a camera, the epipolar geometry is as it were checked on the off chance that the watched scene is unbending.

3.2. Fundamental Matrix

The elemental matrix expresses the epipolar interface within the theory that the inner parameters of the video sensors are obscure. Thus, for a point q1 of the picture of the primary sensor, able to calculate the line q1 on which the comparing perception is found within the moment sensor (figure.04) [5]:

$$l2 \sim F\dot{q}1 \text{ with } F=\begin{bmatrix} f11 & f12 & f13 \\ f21 & f22 & f23 \\ f31 & f32 & f33 \end{bmatrix} \quad (1)$$

l2 is called epipolar line associated with q1. Moreover, if two observations q1 and q2 correspond to the same point of space, they check:

$$\dot{q}_2^T F \dot{q}_1 = 0 \Rightarrow l2\ T\ \dot{q}_1 = 0 \Rightarrow \dot{q}_1 \in l2 \quad (2)$$

$$\dot{q}_2^T F \dot{q}_1 = 0 \Rightarrow l1 \dot{q}_2^T = \mathbf{0} \Rightarrow \dot{q}_2 \in l1 \quad (3)$$

This relationship permits assessing the basic network from the 2D affiliation between two photographs. In reality, one can calculate with 8 focuses [06] or from 7 focuses beneath a few suspicions [07].

In each image, a dot plays a particular function. It is the two epipoles e1 and e2. They correspond to the projection in the photo of the optical center of the second camera. The epipoles have two nice features. They define the core of F:

$$F\,\dot{e}1 = F\,\dot{e}2 = 0 \qquad (4)$$

In addition, the epipoles correspond to the points of intersection of all epipolar lines in each of the photos.

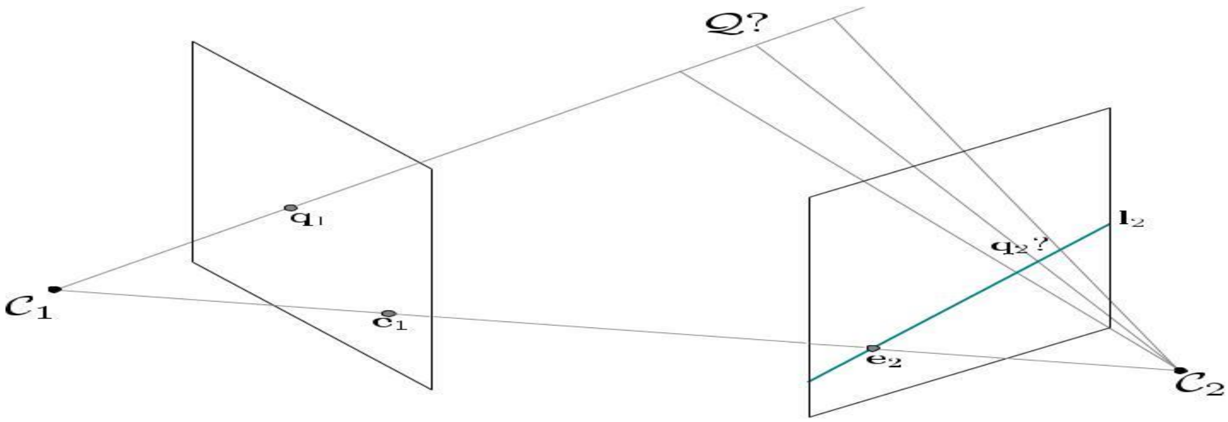


Fig. 04: Epipolar geometry

3.3. The Essential Matrix

The essential matrix E is capable of being seen as the particular case of the fundamental matrix in the hypothesis that the calibration of the video sensors (K1 and K2) is known. The link between the essential matrix and the fundamental matrix is as follows:

E~K2TFK1 (5)

Equation (3) becomes in this case:

$$\dot{q}_2^T (K_2^{-T}\, E\, K_1^{-1}) \dot{q}_1 \qquad (6)$$

Where K_2^{-T} is the inverse transpose of K2. For the estimation of the essential matrix.

The essential matrix can have several uses. Indeed, there is a link between the essential matrixes of two video sensors (C1, C2) to the relative displacement between these two. Relative displacement is defined by: $(R1\rightarrow2 , T1\rightarrow2)$

The relationship between the essential matrix and relative displacement E, $R1\rightarrow2$ and $T1\rightarrow2$ is written: $E= [T1 \rightarrow 2]_x \, R1\rightarrow2$ (7)

Where $[T]_x$ is the antisymmetric matrix constructed from the T vector, namely:

$$[T]_x = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \qquad (8)$$

An estimate of the essential matrix E then allows finding the relative displacement between the two video sensors.

3.4. Camera Placement and Relative Displacement

The reason of this segment is to formalize the idea of relative uprooting between the two video sensors and the related documentations. We have seen, the laying of these can be known as a alter of reference between the world point of interest and the points of interest joined to the video sensors:

$$Q\,C1 = (R1 \quad T1)\,\dot{Q}_w \qquad (9)$$

$$Q\,C2 = (R2 \quad T2)\,\dot{Q}_w \qquad (10)$$

With:

Qw is a 3D point expressed in the world landmark

Qc1 and Qc2 are the respective Qw coordinates in the C1 and C2 video sensor markers.

$(R1\rightarrow2,\ T1\rightarrow2)$ is the relative displacement between the two video sensors, that is, the transformation to change from the mark associated with C1 to that linked to C2:

$QC2=(R1^{\rightarrow}2\ T1^{\rightarrow}2)\dot{Q}_{c1}$ (11)

Equations (9) and (10) give:

$R1^{\rightarrow}2 = R2\ R_1^T$ (12)

$T1^{\rightarrow}2 = T2 - R2\ R_1^T T1$ (13)

Curiously ponder based on computer vision which is virtual or expanded reality. The number one step in our system is to utilize information to discover a to begin with estimation of the camera's area. The position and introduction of the camera is in this manner related with each picture within the grouping. The second step is to associate the pictures and the 3D demonstrate in arrange to get correct shots of the grouping camera, for each picture of the video. The advanced camera is initialized with the positions given either by GPS or other, the projection of the demonstrate is recorded with the pictures by changing the position and introduction of the virtual camera [08]

Increased reality (AR) may be a technique that coexists a virtual world within the genuine world (Fig. 05, 06, 07). The client will be able to see made formats that coincide with a genuine environment. Increased reality is intrigue and is based on flag preparing, fake vision, picture blend and human-application interfacing. This innovation has experienced a solid improvement in recent a long time, coordination a computerized camera and adequately capable equipment to form the virtual environment [09].

The visual rendering of increased reality can be circuitous, that's through the projection screen or coordinate by means of a videoheadset or semi-transparent glasses [10]. The reason of counterfeit vision AR is, subsequently, to reinforce our vision of the genuine environment by including avatars that are likely not noticeable by the human eye. And after that AR is characterized as an environment that can combine genuine scenes with virtual avatars [11]. The composition must be intelligently and in genuine time where the client sees virtual objects at the same time with the genuine environment through which he is advancing. The real-time commitment [12] makes AR crucial for any change, whereas in post generation, different picture handling and vision program permit changing genuine video film. Thus, the ought to embed real-time character to properly justify the value of the AR.

In expanding a scene, the practical impact is fundamental. In any case, the interaction of the client in genuine time with virtual avatars is similarly critical. He seem select them, move them additionally destroy them by employing a few instruments. All this must consider the proper enrollment of virtual and genuine objects, and their occultation.

4. Results and Discussion

By and large, to extend a scene, one must have a camera mostly fitted to a landmark. Its calibration points to set, up geometrically, its optical characteristics and its area and introduction. The scene too has its claim clear reference point, where the positions of certain genuine components will essentially be known. Expansion medicines must consider the inactivity times [13], factors and huge, of the rebellious that make up our framework. Their dissimilarities (unwavering quality, recurrence, etc...) require a few spatial or transient rectifications. The addition of geometric shapes modeled in 3D requires markers to be found within the genuine scene. Once found, they will be misused as inlay markers. The reason of 'tracking' is to discover their shapes and positions within the scene picture mostly at a 3D numerical show of each design utilized.

The trim of the 3D geometrques shapes is at that point done concurring to the standards of geometric projections. The assessment of the framework H (planar homogenography) is done for each photo of the scene. It compares to the arrangement of the condition:

$[x\ y\ w]t = H\ [x'y'w']t$

(14)

In the case of a 2D plane projection, this can be solved by the singular values method. In the equation (x',y',w) are homogeneous coordinates of an X point of the avatar relative to its landmark and (x,y,w) are homogeneous coordinates in the landmark of the photo. The resolution of this equation requires the knowledge of four points in the photo landmark [14].

On the contrary to virtual reality, augmented reality will not accentuate the immersion of the user in the virtual world created because the real world comes into play here, just as on the interaction between the user and the virtual environment, but will in particular be based on what is called tracking. Tracking is the alignment of the virtual to the real and this in real time; it is a vital element in augmented reality because it will result in the homogeneity of the information given to the user. If a 3D geometric shape is displayed or moved, it will follow it (figure.03).

4.1. Real and Virtual Camera Alignment

Taking as an example the (figure.05, 06, 07) as our test environment, the first problem is to match the perspective of the geometric form with that of the actual sequence. This problem or error is known through the virtual and real camera alignment name.

To process it, we must first get hold of the properties of the real camera that generated the observation, after calculating the synthetic photos using a virtual camera recovering these properties.

Cube (Fig. 05): étude théorique sur papier (méthode classique) et application de la réalité augmentée sur le modèle cube 3D.

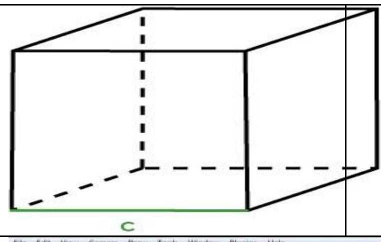
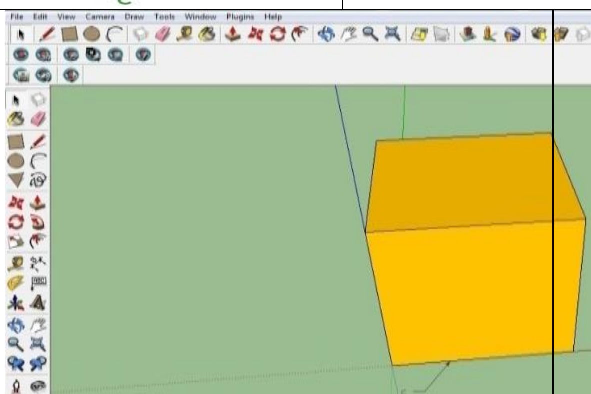
	The area of the cube: AB= c2 AL=4c2 AT=6c2	The volume of the cube: V=c3
		

Fig. 05: étude expérimentale, modélisation 3D et application de la réalité augmentée sur le modèle cube 3D.

Prisme (Fig. 06): étude théorique sur papier (méthode classique) et application de la réalité augmentée sur le modèle cube 3D.

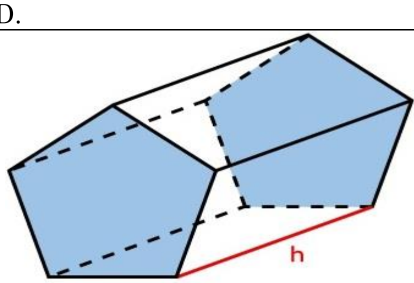
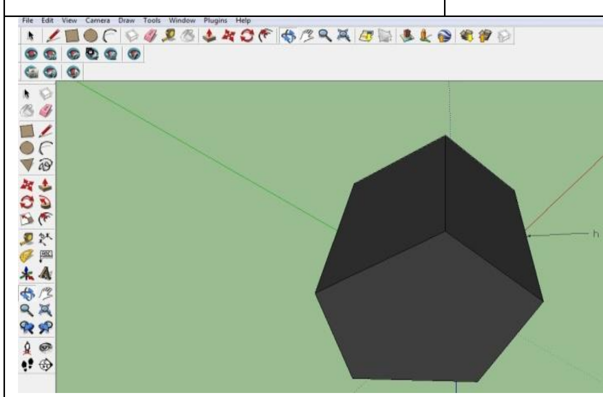
	**The area of the prisms:** Ab= formula associated with the figure AL=Pb×h AT=AL+2Ab	**The volume of the prisms:** V=Ab×h
		

Fig. 06: étude expérimentale, modélisation 3D et application de la RA sur le modèle prisme 3D

- ✓ **PYRAMID Fig. 07:** étude théorique sur papier (méthode classique) et application de la réalité augmentée sur le modèle cube 3D.

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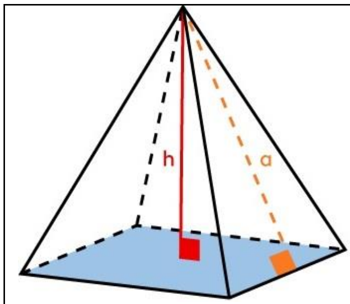
<td></td>
<td>The area of the pyramids:
Ab= formula associated with the figure
$A_L = \frac{Pb \times a}{2}$
AT=AL+Ab</td>
<td>The volume of the pyramids:
$V = \frac{A_b}{3} \times h$</td>
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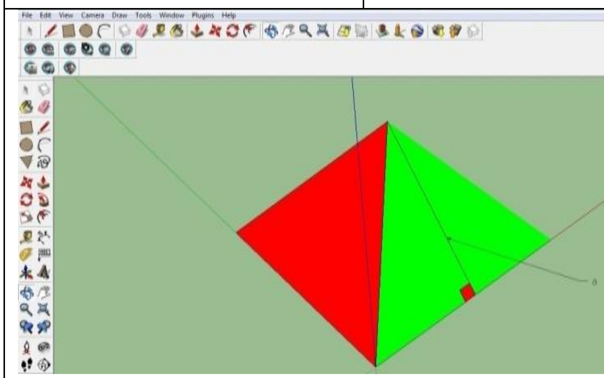
<td colspan="2"></td>
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Fig. 07: étude expérimentale, modélisation 3D et application de la RA sur le modèle prisme 3D

5. Conclusion

Learning mathematics through augmented reality offers an innovative and immersive approach that can significantly improve the educational experience. In conclusion, here are some key points to remember:

- ✓ Increased student engagement: Augmented reality makes math concepts more tangible and interactive, which captures students' attention and encourages them to be more engaged in their learning.
- ✓ Visualization of abstract concepts: Often abstract mathematical concepts become more concrete through 3D visualization and virtual manipulation of objects, which facilitates understanding and retention.
- ✓ Experiential learning: Augmented reality allows students to explore and experiment with mathematical concepts in virtual environments, promoting a hands-on, experiential approach to learning.
- ✓ Adaptability and personalization: Augmented reality applications can be tailored to students' individual needs, providing personalized learning experiences that promote greater understanding of mathematical concepts.
- ✓ Integrating Emerging Technologies: Integrating augmented reality into mathematics education prepares students to work with emerging technologies and develop essential digital skills for their future.

In summary, the use of augmented reality in mathematics learning has many benefits in terms of engagement, understanding and preparation for future technological challenges, making it a promising approach for mathematics education.

In conclusion, the application of augmented reality in learning geometric shapes represents a significant advance in mathematics teaching. This technology offers an immersive and interactive approach that allows students to visualize and manipulate abstract concepts in a concrete way, making them easier to understand and learn.

Through augmented reality, geometric shapes become tangible objects that students can explore in their real-world environment, making geometric concepts more accessible and engaging. Additionally, this technology provides personalized learning opportunities by tailoring activities and exercises to students' individual needs, promoting a more effective and engaging learning experience.

By integrating augmented reality into the teaching of geometric shapes, educators can enrich math lessons, stimulate students' interest in geometry, and promote the development of problem-solving and spatial thinking skills. Ultimately, the application of augmented reality on geometric shapes opens new avenues for making mathematics learning more interactive, accessible and enriching for students of all levels.

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